

博士生资格考试《代数与数论方向》考试大纲 (From 2025 Fall)

一、前言

为了进一步提高清华大学求真书院博士研究生的质量, 特为代数与数论方向博士生资格考试制订本大纲。考试对象为参加清华大学博士生入学考试或资格考试的考生。

二、《代数与数论方向》考试大纲 From 2025 Fall

Basic (70 points)

- Group theory: Sylow theorems, p -groups, solvable groups, free groups.
- Rings and modules: tensor products, determinants, Jordan canonical form, PID's, UFD's, polynomial rings.
- Field theory: splitting fields, separable and inseparable extensions.
- Galois theory: Fundamental theorems of Galois theory, finite fields, cyclotomic fields.
- Homological algebra: exact sequences, snake and five lemmas, projective, injective, and flat modules, complexes, (co)homology.
- Commutative algebra: localizations, Hilbert's basis theorem, integral extensions, radicals of ideals, Zariski topology and Hilbert's Nullstellensatz, Dedekind rings, discrete valuation rings.
- Representations of Finite Groups: character theory, induced representations, structure of the group ring.
- Basics of Lie groups and Lie algebras: exponential map, nilpotent and semi-simple Lie algebras and Lie groups, structure of semisimple Lie groups/Lie algebras and their representations.

Advanced (30 points, choose 2 from 6 problems)

Number Theory:

- Decomposition of primes in number fields (decomposition group, inertia group, Frobenius, relative different and discriminant)
- Finiteness theorems (ideal class groups, class number, and Dirichlet's unit theorem)
- p -adic methods (p -adic fields, non-archimedean valuations, completion, Hensel's lemma, Newton polygons)
- Analytic methods (Dedekind zeta functions and the class number formula)

Algebraic Geometry:

- Varieties (dimension, irreducibility, smooth and singular points, function fields and rational maps, proper morphisms, differentials and derivations)
- Segre and Veronese embeddings, Grassmannians and Plücker embeddings
- Hilbert polynomials and Bézout's theorem

- Algebraic curves (genus, Riemann-Roch theorem, Riemann-Hurwitz theorem)

Geometric representation theory:

- Basic of category $\mathcal{O}$ (Weyl character formula, Verma modules, Harish-Chandra isomorphism, block decomposition, projective module)
- Geometry of flag variety (Bruhat decomposition, Plücker embedding)
- Hecke algebra

三、《代数与数论方向》考核要求目标

对代数与数论方向考试内容有很好的理解和掌握，包括基本定义、性质、定理及其证明思想、内在联系、主要技巧、主要理论的基本应用等。能应用基本理论、方法和技巧证明或解答相应的题目。

四、考试方式

闭卷，笔试，试卷满分 100 分

五、参考书目：

- M. Artin, Algebra
- D. S. Dummit and R. M. Foote, Abstract Algebra
- S. Lang, Algebra
- M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra
- J.-P. Serre, Representations of Finite Groups
- P. Etingof et al, Introduction to Representation Theory
- W. Fulton and J. Harris, Representation Theory: A First Course
- J. E. Humphreys, Introduction to Lie algebras and Representation theory
- D. A. Marcus, Number Fields
- Z. I. Borevich and I. Shafarevich, Number Theory
- S. Lang, Algebraic Number Theory
- J. Neukirch, Algebraic Number Theory
- E. Looijenga, Algebraic varieties
- J. Harris, Introduction to Algebraic Geometry
- J. E. Humphreys, Representations of semisimple Lie algebras in the BGG category $\mathcal{O}$