

Probability & Statistics

Syllabus

Probability:

- Random variable, Expectation, Independence
- Variance and covariance, correlation, moment
- Various distribution functions
- Multivariate distribution
- Characteristic function, Generating function
- Various modes of convergence of random variables
- Bayes formula, Conditional probability
- Conditional expectation given a sigma-algebra
- Laws of large numbers
- Central limit theorems
- Martingales
- Markov chains
- Basic properties of Poisson processes
- Basic properties of Brownian motion
- Basic properties of random walks

Statistics:

- Distribution Theory. Families of continuous distributions: normal, chi-sq, t, F, gamma, beta; Families of discrete distributions: multinomial, Poisson, negative binomial; Exponential Families; Distributions of functions of a Random Variable.
- Properties of a Random Sample: order statistics, sample mean, variance, median and quantiles; the Delta Method.
- Data Reduction: sufficient statistics, minimal sufficient statistics, ancillary statistics, complete statistics, Basu's Theorem.
- Estimation: Parameter estimation, method of moments, maximum likelihood estimation, criteria for evaluation of estimators, Fisher information and its use, confidence interval.
- Testing: Null and alternative hypotheses, simple and composite hypotheses, type I and type II errors, power, uniformly most powerful test, likelihood ratio test, Neyman-Pearson Lemma, Karlin-Rubin Theorem, generalized likelihood ratio test.
- Bayesian Statistics: Prior, posterior, conjugate priors, Bayesian estimator.
- Large sample properties: Consistency, asymptotic normality, chi-sq approximation to likelihood ratio statistic.
- Linear model theory: least squares estimation, estimability, Gauss-Markov, inference on parameters (point and interval estimations and testing), prediction, confidence region and simultaneous confidence intervals, model comparison and selection, and goodness of fitness test under normal errors.

Written examination

Reference:



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- [5] 陈家鼎, 孙山泽, 李东风, 刘力平: 数理统计学讲义, 高等教育出版社, 2006.
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