

Analysis & Differential Equations

Syllabus

Real & Functional Analysis:

- Measure theory: measure space, outer measure, Lebesgue measure, integration on measure space, convergence theorems for integrals, signed measure, Radon-Nikodym theorem, Lebesgue decomposition theorem, product of measure space, Fubini theorem.
- L^p space: Riesz-Fisher theorem, duality of L^p space, Jensen inequality.
- Differentiation theorems: functions of bounded variation, absolute continuous functions, covering theorems, Lebesgue differentiation theorem, Newton-Leibniz formula.
- Borel measure on topological spaces: Radon measure, Riesz representation theorem.
- Theory of Banach spaces: Hahn-Banach Theorem, open mapping theorem, uniform boundedness theorem, closed graph theorem, dual space, weak and weak *-convergence.
- Theory of Hilbert spaces: Riesz representation theorem, Lax-Milgram theorem.
- Operator theory: basic properties of compact operators, Riesz-Fredholm theory, spectrum of compact operators, Fredholm operators.
- Fourier series: convergence of Fourier series, Cesaro and Abel summability, applications of Fourier series.

Complex Analysis:

- Holomorphic functions: complex integrals, Cauchy's theorem, Cauchy integral formula, Morera's theorem.
- Meromorphic functions: zero and poles, residues, the argument principle.
- Harmonic functions: the mean value property; the reflection principle; Dirichlet's problem.
- Series and product developments: power series and Laurent series, partial fractions expansions, Weierstrass infinite products, Hadamard's factorization theorem.
- Special functions: the Gamma function, the Zeta function, elliptic functions.
- Conformal mappings: linear fractional transformations, Schwarz lemma, Riemann mapping theorem, Picard theorems.
- Basics of Riemann surfaces.

Ordinary & Partial Differential Equations:

- Ordinary differential equations: explicit solutions of simple equations, existence and uniqueness theory for initial value problems, systems of linear ordinary differential equations, phase space analysis, stability theory, Lyapunov method.
- Boundary value problems for second order differential equations: Sturm-Liouville theory, separation of variables for certain partial differential equations.
- First order partial differential equations: transport equations, method of characteristics, Burgers' equation, Hamilton-Jacobi equation, conservation laws.
- Second order linear elliptic equations: fundamental solution for Laplace operator, Green's functions, weak solution, existence and uniqueness of solutions of Dirichlet and Neumann boundary value problem, energy method, maximal principles, Harnack's inequality.
- Second order linear parabolic equations: fundamental solution to heat equation, energy method,

weak solutions, the existence and uniqueness of solutions to initial value problems and initial-boundary value problems, energy method, maximal principles, semigroup theory.

·Second order linear hyperbolic equations: fundamental solutions to wave equations, weak solutions, the existence and uniqueness of solutions to initial value problems and initial-boundary value problems, energy method, propagation of solutions, semigroup theory.

·Distribution theory: test functions and distributions, operations on distributions, product with smooth functions, differentiation, tensor product, convolutions, pullback and pushforward under smooth mappings. Schwartz space, tempered distributions and Fourier transform.

·Sobolev spaces: approximation by smooth functions, extensions, traces, Sobolev embedding theorem, Poincare's inequality.

Written examination

Reference:

- [1] Walter Rudin, Real and Complex Analysis, Third Edition, McGraw-Hill Book Co., 1987.
- [2] Elias M. Stein and Rami Shakarchi, Real Analysis. Measure theory, integration, and Hilbert spaces, Princeton University Press, 2005.
- [3] Gerald B. Folland, Real analysis. Modern Techniques and Their Applications, Second Edition, John Wiley & Sons, 1999.
- [4] Peter Lax, Functional Analysis, John Wiley & Sons, 2002.
- [5] Elias M. Stein and Rami Shakarchi, Fourier Analysis, An Introduction, Princeton University Press, 2003.
- [6] Lars Ahlfors: Complex Analysis-an introduction to the theory of analytic functions of one complex variable. Third Edition, McGraw-Hill Book Co. 1978.
- [7] Elias M. Stein and Rami Shakarchi, Complex Analysis, Princeton University Press, 2003.
- [8] Wolfgang Walter, Ordinary Differential Equations, Springer-Verlag, 1998.
- [9] Lawrence Craig Evans, Partial Differential Equations, Second edition, AMS, 2010.
- [10] Lars Hormander, The Analysis of Linear Partial Differential Operators, vol. I, Springer-Verlag, 1983.