Problems for GR qualifying examination

Solutions should include sufficients details and explanations.

We consider spacetime metrics with signature $-+++$.

Problem 1: Analysis of null geodesics in the Schwarzshild spacetime with line element

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad f = 1 - 2M/r\,.$$

Let $\xi^a = (\partial_t)^a$, $\psi^a = (\partial_\phi)^a$ be the static and axial Killing fields, respectively. The energy E and azimuthal angular momentum L of a null geodesic with 4-velocity u^a are given by

$$E = -\xi^a u_a, \quad L = \psi^a u_a$$

a) Show that E and L are conserved.

10 pts.

b) Write an expression for the energy of an equatorial null geodesic ($\theta = \pi/2$) in terms of its azimuthal angular momentum and radial velocity.

10 pts.

Problem 2: Consider the paraboloid P with equation $x^2 + y^2 + z^2 - t^2 = \Lambda^2$ in Minkowski space with metric $-dt^2 + dx^2 + dy^2 + dz^2$.

a) Show that the induced metric on P is Lorentzian.

10 pts.

b) Calculate the Einstein tensor of P.

20 pts.

Problems for QFT qualifying examination

1) The Dirac Field.

i) Show that the Dirac equation is Lorentz invariant.

10 pts.

ii) Compute the Dirac propagator

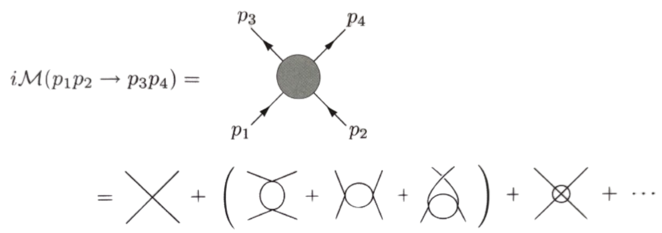
$$iS_{\alpha\beta}(x) \equiv \langle 0| T \left[\Psi_\alpha(x) \overline{\Psi}_\beta(0) \right] |0\rangle,$$

and show that it can be written as

$$iS(x) = i \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \frac{i}{\not{p} - m + i\epsilon} \ .$$

15 pts.

2) Coupling constant renormalization in φ^4-theory. The two-particle scattering amplitude in φ^4-theory is given by



$$= -i\lambda_P + (-i\lambda_P)^2 \left[iV(s) + iV(t) + iV(u) \right] + 4! iC,$$

where the last term is the counterterm. The renormalization condition is given by

$$i\mathcal{M} = -i\lambda_P \quad \text{at} \quad s = 4m_P^2, \ t = u = 0.$$

Compute $V(p^2)$ using dimensional regularization and Feynman parameters. Fix C to second order in λ_P this way.

25 pts.